Group 6

Implementation

Padding

- What is Padding?
- Types of padding.
- Comparison

What is padding

• Padding extends the boundaries of an image to avoid undefined operations when parts of a kernel (mask) lie outside the border of the image during filtering. In general, if the kernel is of size m*n, we need (m-1)/2 rows at the top and bottom and (n-1)/2 columns at the right and left and the values these new cells will take depends on the padding technique.

Types of padding

- Zero padding
- Mirror (or symmetric) padding
- Replicate padding

Mirror padding

- Values outside the boundary of the image are obtained by mirror-reflecting the image across its border.
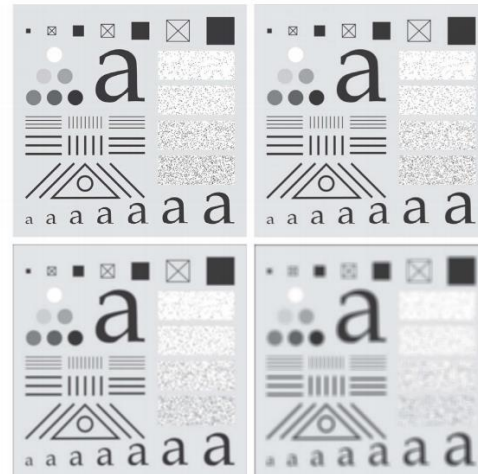
Replicate padding

- Values outside the boundary are set equal to the nearest image border value. It is useful when the areas near the border of the image are constant.

1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
6	6	6	7	8	9	10	10	10
11	11	11	12	13	14	15	15	15
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20

Comparison

- Zero Padding: When zero (black) padding is used, the net result of smoothing at or near the border is a dark gray border that arises from including black pixels in the averaging process. Using the 11*11 kernel resulted in a more prominent dark border. The result with the 21*21 the kernel shows significant blurring of all components of the image, including a darker boarder. The two other approaches to padding tend to solve the dark-border problem that results from zero padding.

- Mirror Padding: mirror padding does duplicate the details of the image on the edges which makes it more applicable when the areas near the border contain image details.

- Replicate Padding: It duplicate the only pixel on the edge which makes it useful when the areas near the border of the image are constant.
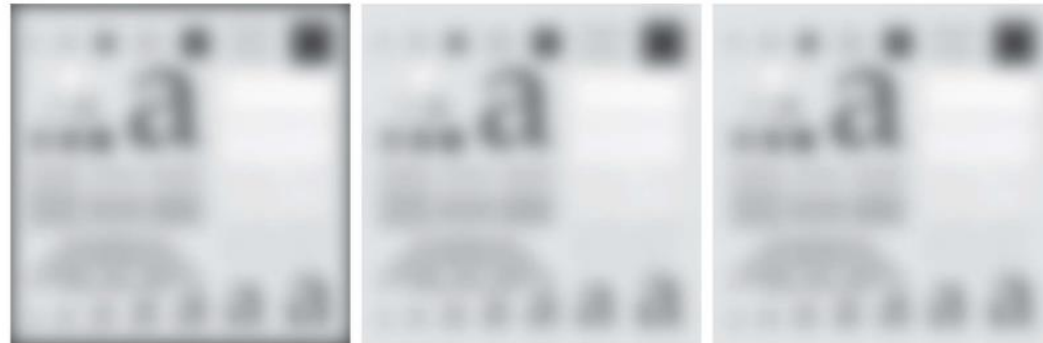
Convolution and Correlation Filters

- **Spatial Filters**

- **Spatial Correlation Filter**

- **Procedure**

- **properties**

Spatial filter

- Spatial filtering modifies an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors. If the operation performed on the image pixels is linear, then the filter is called a linear spatial filter. Otherwise, the filter is a nonlinear spatial filter.

Spatial correlation filter

Spatial filtering modifies an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors.

Correlation consists of moving the center of a kernel over an image and computing the sum of products at each location.

$$(w \star f)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

Where w (s, t) is the kernel and f (x, y) is the padded original image. And the left side express the correlation operation between kernel w of size of m*n and image f (x, y).

Spatial convolution filter

- The mechanics of spatial convolution are the same, except that the kernel is rotated by 180°. Thus, when the values of a kernel are symmetric about its center, correlation and convolution yield the same result.

$$(w \star f)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t)$$

Where w (s, t) is the kernel and f(x, y) is the padded original image. And the left side express the convolution operation between kernel w of size of m*n and image f (x, y).

Notice that the minus signs indicate one of the functions is rotated by 180°.

Procedure

For both two filters, we begin with padding the original image (f) as following.

Padded f

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	**1**	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Origin f

0	0	0	0	0
0	0	0	0	0
0	0	**1**	0	0
0	0	0	0	0
0	0	0	0	0

w

1	2	3
4	5	6
7	8	9

(a)

(b)
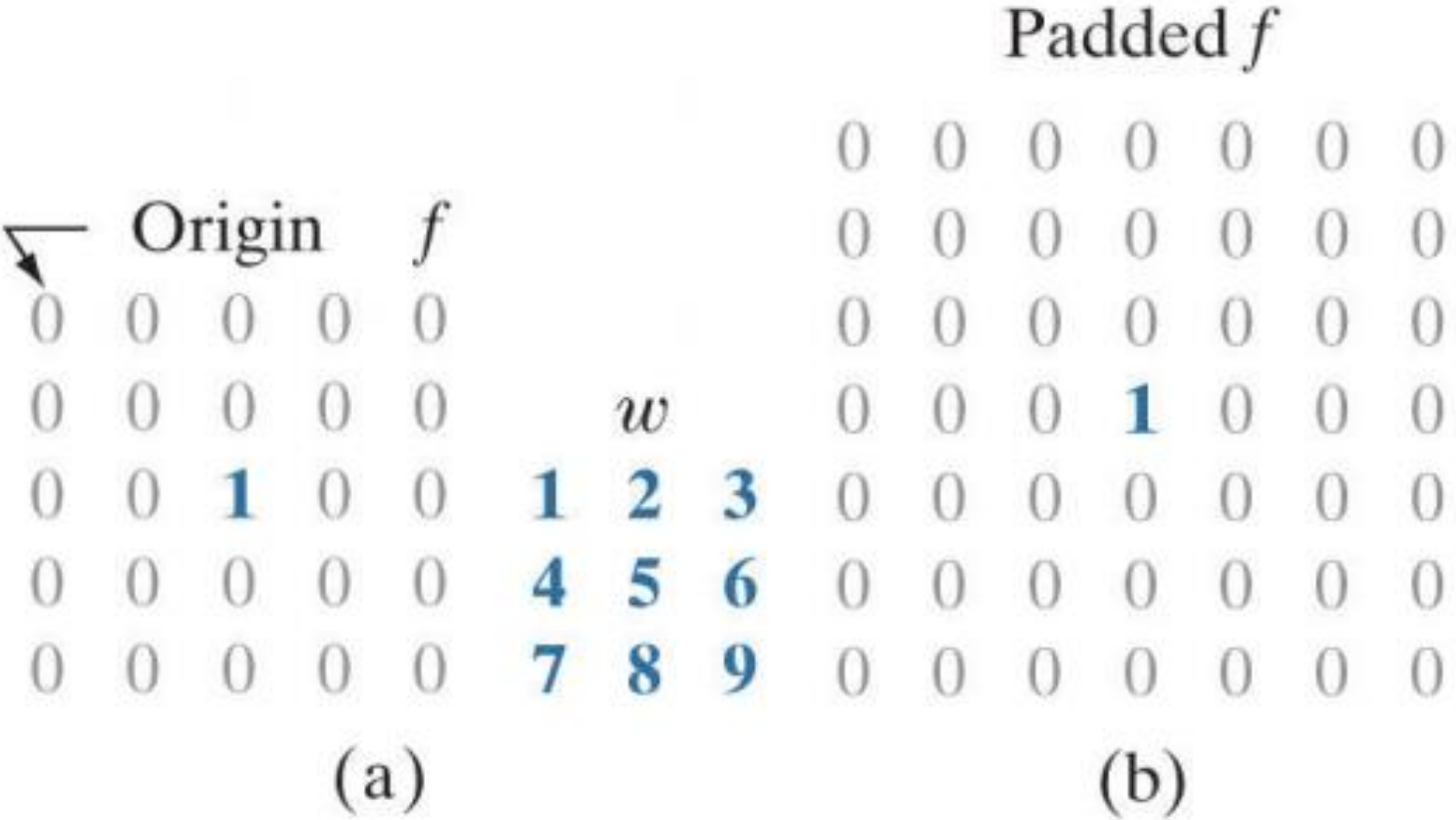
For spatial correlation, we begin moving the filter kernel along the image calculating the sum of products as following.

Initial position for w

```
1  2  3  0  0  0  0
4  5  6  0  0  0  0
7  8  9  0  0  0  0
0  0  0  1  0  0  0
0  0  0  0  0  0  0
0  0  0  0  0  0  0
0  0  0  0  0  0  0
```

(c)

Correlation result

```
0  0  0  0  0
0  9  8  7  0
0  6  5  4  0
0  3  2  1  0
0  0  0  0  0
```

(d)

Full correlation result

```
0  0  0  0  0  0  0
0  0  0  0  0  0  0
0  0  9  8  7  0  0
0  0  6  5  4  0  0
0  0  3  2  1  0  0
0  0  0  0  0  0  0
0  0  0  0  0  0  0
```

(e)

For spatial convolution, we do the same except we rotate the kernel by 180° which means in 2d that the kernel is flipped about one axis and then the other.

Rotated w

9	8	7	0	0	0	0
6	5	4	0	0	0	0
3	2	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

(f)

Convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

(g)

Full convolution result

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	1	2	3	0	0
0	0	4	5	6	0	0
0	0	7	8	9	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

(h)

Properties

Some fundamental properties of convolution and correlation.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

Sometimes an image is filtered (i.e., convolved) sequentially, in stages say Q stage. Because of the commutative property of convolution, this multistage filtering can be done in a single filtering operation, as following.

$$w = w_1 \star w_2 \star w_3 \star \cdots \star w_Q$$

Note that We cannot do the same reduction for correlation because it is not commutative.